Assignment 8

CSI 4336

Due November 29, 2016

Submitting your assignment

All written portions of the assignment should be prepared in \LaTeX.

Submit this assignment on the due date in two ways: by email (before class) and in hardcopy (at the beginning of class). The printed copy should not contain any programming code. Proofread your document for style before submitting it.

Send the email to hamerly@cs.baylor.edu with the subject “CSI 4336 assignment X”, where X is the assignment number (e.g. 0). The email should have one attachment (in plain text, .zip, or .tar.gz format) containing:

- the .tex document you wrote named “lastname.tex” (where ‘lastname’ is your last name),
- any additional files used in your \LaTeX document, named “lastname_fig1.pdf” (or similar), and
- all source code used for any programs.

NP-completeness proofs

Prove the following languages are NP-complete. To show this, you must show both that the languages are in NP, and that some known NP-complete language is poly-time reducible to the language.

For these problems, you are limited to the following NP-complete languages for the reductions:

\[
\begin{align*}
3\text{SAT} & = \{ \langle \phi \rangle \mid \phi \text{ is a satisfiable 3-CNF sentence } \} \\
\text{CLIQUE} & = \{ \langle G, k \rangle \mid \text{graph } G \text{ has a } k\text{-clique} \} \\
\text{HAMCYCLE} & = \{ \langle G \rangle \mid \text{graph } G \text{ has a Hamiltonian cycle } \} \\
\text{HAMPATH} & = \{ \langle G, s, t \rangle \mid \text{graph } G \text{ has a Hamiltonian path from } s \text{ to } t \} \\
\text{SUBSET-SUM} & = \{ \langle S, t \rangle \mid S \text{ is a multiset of integers where some } U \subseteq S \text{ sums to } t \} \\
\text{TSP} & = \{ \langle G, k \rangle \mid \text{weighted graph } G \text{ has a tour with cost } \leq k \} \\
\text{VC} & = \{ \langle G, k \rangle \mid \text{graph } G \text{ has a vertex cover of size } \leq k \}
\end{align*}
\]

You may email me for a hint on either problem, and I will give you the problem that I think should be used for the reduction, but it will cost you two points per hint.

1. (10 points) Consider the following language:

\[
\text{QUAD3SAT} = \{ \langle \phi \rangle \mid \phi \text{ is a 3-CNF formula having at least 4 different satisfying assignments} \} 
\]
For example, the following string is in QUAD3SAT:

$$(x \lor x \lor \bar{x}) \land (y \lor y \lor \bar{y})$$

because there are four different assignments that satisfy it (00, 01, 10, and 11). Prove that QUAD3SAT is NP-complete.

2. (15 points) Consider the following language:

$$\text{LCS} = \{ \langle G_1, G_2, k \rangle \mid G_1 \text{ and } G_2 \text{ are graphs that have isomorphic subgraphs with } k \text{ edges each} \}$$

Prove that LCS is NP-complete.

Recall the following definitions:

- A subgraph $G' = (V', E')$ of a graph $G = (V, E)$ has the following properties: $V' \subseteq V$ and $E' \subseteq E$.
- Two graphs $G = (V, E)$ and $H = (V', E')$ are isomorphic if $|V| = |V'|$ and $|E| = |E'|$ and there is a function $f : V \rightarrow V'$ such that $(u, v) \in E \iff (f(u), f(v)) \in E'$.

3. (15 points) Consider the following language:

$$\text{IST} = \{ \langle G, T \rangle \mid G \text{ is a graph with a spanning tree isomorphic to tree } T \}$$

Prove that IST is NP-complete.