Assignment 8

CSI 4336

Due November 15, 2018

Submitting your assignment

All written portions of the assignment should be prepared in LaTeX.

Submit this assignment by the due date in two ways: by email (before class) and printed (at the beginning of class). Don't put any code in the printed copy. Proofread your document for style before submitting it.

Send the email to `hamerly@cs.baylor.edu` with the subject "CSI 4336 assignment X" (where X is the assignment number). The email should have one attachment (plain text, .zip, or .tar.gz format) containing:

- the .tex document you wrote named "lastname.tex" (where 'lastname' is your last name),
- a compiled .pdf from the .tex document named "lastname.pdf" (where 'lastname' is your last name),
- any additional files used in your LaTeX document, named "lastname_fig1.pdf" (or similar), and
- all source code used for any programs.

NP-completeness proofs

Prove the following languages are NP-complete. To show this, you must show both that the languages are in NP, and that some known NP-complete language is poly-time reducible to the language.

For these problems, you are limited to the following NP-complete languages for the reductions:

$$\begin{aligned}
\text{3SAT} &= \{\ \langle \phi \rangle \mid \phi \text{ is a satisfiable 3-CNF sentence } \} \\
\text{CLIQUE} &= \{\ \langle G, k \rangle \mid \text{graph } G \text{ has a } k\text{-clique } \} \\
\text{HAMCYCLE} &= \{\ \langle G \rangle \mid \text{graph } G \text{ has a Hamiltonian cycle } \} \\
\text{HAMPATH} &= \{\ \langle G, s, t \rangle \mid \text{graph } G \text{ has a Hamiltonian path from } s \text{ to } t\ \} \\
\text{SUBSET-SUM} &= \{\ \langle S, t \rangle \mid S \text{ is a multiset of integers where some } U \subseteq S \text{ sums to } t\ \} \\
\text{TSP} &= \{\ \langle G, k \rangle \mid \text{weighted graph } G \text{ has a tour with cost} \leq k\ \} \\
\text{VC} &= \{\ \langle G, k \rangle \mid \text{graph } G \text{ has a vertex cover of size} \leq k\ \}
\end{aligned}$$

You may email me for a hint on either problem, and I will give you the problem that I think should be used for the reduction, but it will cost you two points per hint.

1. (10 points) Consider the following language:

   $$\text{QUAD3SAT} = \{\ \langle \phi \rangle | \phi \text{ is a 3-CNF formula having at least 4 different satisfying assignments}\}$$

For example, the following string is in QUAD3SAT:

$$(x \vee x \vee \bar{x}) \wedge (y \vee y \vee \bar{y})$$

because there are four different assignments that satisfy it (00, 01, 10, and 11). Prove that QUAD3SAT is NP-complete.

2. (15 points) Consider the following language:

   $$\text{LCS} = \{\ \langle G_1, G_2, k\rangle \mid G_1 \text{ and } G_2 \text{ are graphs that have isomorphic subgraphs with } k \text{ edges each}\ \}$$

   Prove that LCS is NP-complete.

   Recall the following definitions:

   - A subgraph $G' = (V', E')$ of a graph $G = (V, E)$ has the following properties: $V' \subseteq V$ and $E' \subseteq E$.
   - Two graphs $G = (V, E)$ and $H = (V', E')$ are isomorphic if $|V| = |V'|$ and $|E| = |E'|$ and there is a function $f : V \to V'$ such that $(u, v) \in E \Leftrightarrow (f(u), f(v)) \in E'$.

3. (15 points) Consider the following language:

   $$IST = \{\ \langle G, T\rangle \mid G \text{ is a graph with a spanning tree isomorphic to tree } T\ \}$$

   Prove that IST is NP-complete.