Comparing Functions

Notes on Notation

$N = \{0,1,2,3, \ldots \}$

$N^+ = \{1,2,3,4, \ldots \}$

R = Set of Reals

R^+ = Set of Positive Reals

$R^* = R^+ \cup \{0\}$

Comparing $f(n)$ and $g(n)$

Let **f** be a function from **N** to **R**.

O(f) (Big **O** of **f**) is the set of all functions **g** from **N** to **R** such that:

1. There exists a real number **c**>0
2. AND there exists an $\mathbf{n_0}$ in **N**

Such that: $\mathbf{g(n)} \leq \mathbf{cf(n)}$ whenever $\mathbf{n} \geq \mathbf{n_0}$

Notation and Pronunciation

Proper Notation: **g** $\in$ **O(f)**

"g is oh of f"

Also Seen: **g = O(f)**

Big Omega

Let **f** be a function from **N** to **R**.

Ω**(f)** (Big Ω of **f**) is the set of all functions **g** from **N** to **R** such that:

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Such that: $\mathbf{g(n)} \geq \mathbf{cf(n)}$ whenever $\mathbf{n} \geq \mathbf{n_0}$

Big Theta

$\Theta(f) = O(f) \cap \Omega(f)$

“g is of Order f”

$g \in \Theta(f)$

“g is Order f”

Little o and Little Omega

$$o(f) = O(f) - \Theta(f)$$

$$\omega(f) = \Omega(f) - \Theta(f)$$

English Interpretations

- O(f) - Functions that grow no faster than f
- Ω(f) - Functions that grow no slower than f
- Θ(f) - Functions that grow at the same rate as f
- o(f) - Functions that grow slower than f
- ω(f) - Functions that grow faster than f

Limit Formulas

$g \in O(f)$ <u>if</u> $\lim_{n \to \infty} \frac{g(n)}{f(n)} = c,$ <u>for some</u> $c \in R^*$

$g \in \Omega(f)$ <u>if</u> $\lim_{n \to \infty} \frac{g(n)}{f(n)} = \infty$ <u>or</u> $\lim_{n \to \infty} \frac{g(n)}{f(n)} = c > 0$

More Limit Formulas

$$g \in \Theta(f) \quad \underline{\text{if}} \quad \lim_{n \to \infty} \frac{g(n)}{f(n)} = c, \quad \underline{\text{for some}} \quad c \in R^{+}$$

$$g \in o(f) \quad \underline{\text{if}} \quad \lim_{n \to \infty} \frac{g(n)}{f(n)} = 0$$

The Last Limit Formula

$g \in W(f)$ <u>if</u> $\lim_{n \to \infty} \frac{g(n)}{f(n)} = \infty$

Properties

- Transitivity
  - if $f \in O(g)$ and $g \in O(h)$ then $f \in O(h)$
  - Same holds for Θ , Ω, o, and ω
- Anti Symmetry (Sort of ...)
  - $f \in O(g)$ if and only if $g \in \Omega(f)$
  - May replace O with o and Ω with ω

Some More Properties

- Symmetry
  - if $f \in \Theta(f)$, then $g \in \Theta(f)$
- Reflexivity
  - $f \in O(f)$
  - Also true for Θ and Ω
  - Not True for o and ω

And Even More Properties

- Big Theta is an equivalence relation
  - f R g if and only if $f \in \Theta(g)$
- $O(f+g) = O(\max(f,g))$
  - Also true for Θ and Ω

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Such that: **g(n)** ≤ **cf(n)** whenever **n** ≥ $\mathbf{n_0}$

Notation and Pronunciation

Proper Notation: $\mathbf{g} \in \mathbf{O(f)}$

"g is oh of f"

Also Seen: $\boxed{\mathbf{g = O(f)}}$

Big Omega

Let **f** be a function from **N** to **R**.

Ω(f) (Big Ω of **f**) is the set of all functions **g** from **N** to **R** such that:

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- $\Omega(f)$ - Functions that grow no slower than f
- $\Theta(f)$ - Functions that grow at the same rate as f
- $o(f)$ - Functions that grow slower than f
- $\omega(f)$ - Functions that grow faster than f

Limit Formulas

$g \in O(f)$ <u>if</u> $\lim_{n \to \infty} \frac{g(n)}{f(n)} = c,$ <u>for some</u> $c \in R^*$

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More Limit Formulas

$g \in \Theta(f)$ if $\lim_{n \to \infty} \frac{g(n)}{f(n)} = c,$ for some $c \in R^{+}$

$g \in o(f)$ if $\lim_{n \to \infty} \frac{g(n)}{f(n)} = 0$

The Last Limit Formula

$$g \in \mathrm{W}(f) \quad \underline{\text{if}} \quad \lim_{n \to \infty} \frac{g(n)}{f(n)} = \infty$$

Properties

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  - if $f \in O(g)$ and $g \in O(h)$ then $f \in O(h)$
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  - May replace O with o and Ω with ω

Some More Properties

- Symmetry
  - if f $\in$ Θ(f), then g $\in$ Θ(f)
- Reflexivity
  - f $\in$ O(f)
  - Also true for Θ and Ω
  - Not True for o and ω

And Even More Properties

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  - f R g if and only if $f \in \Theta(g)$
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