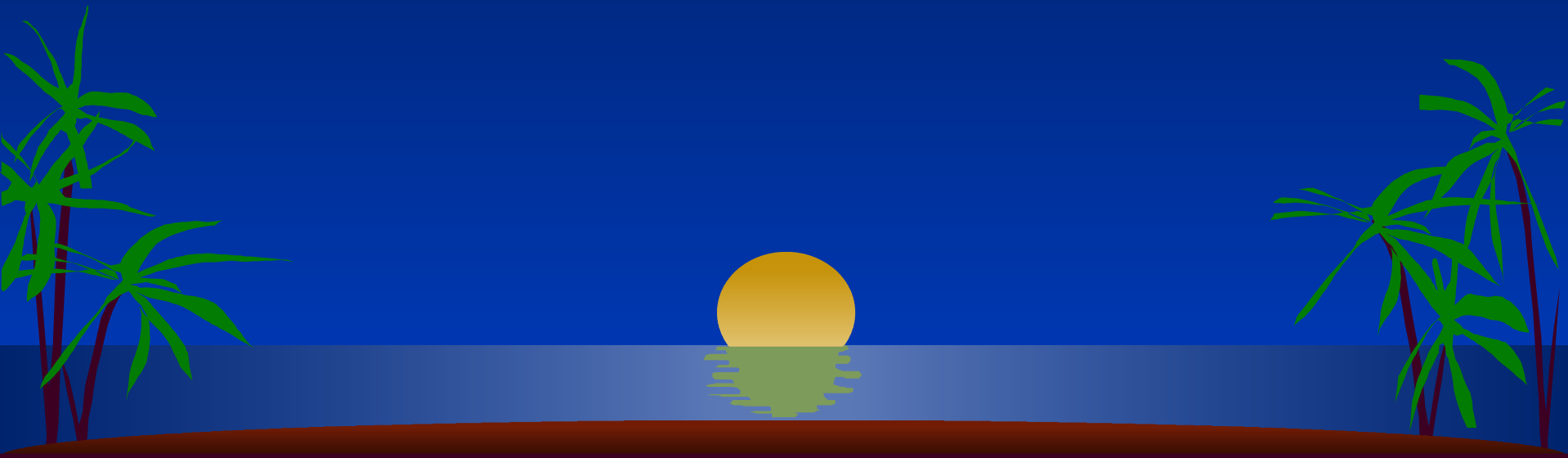


# Comparing Functions



# Notes on Notation

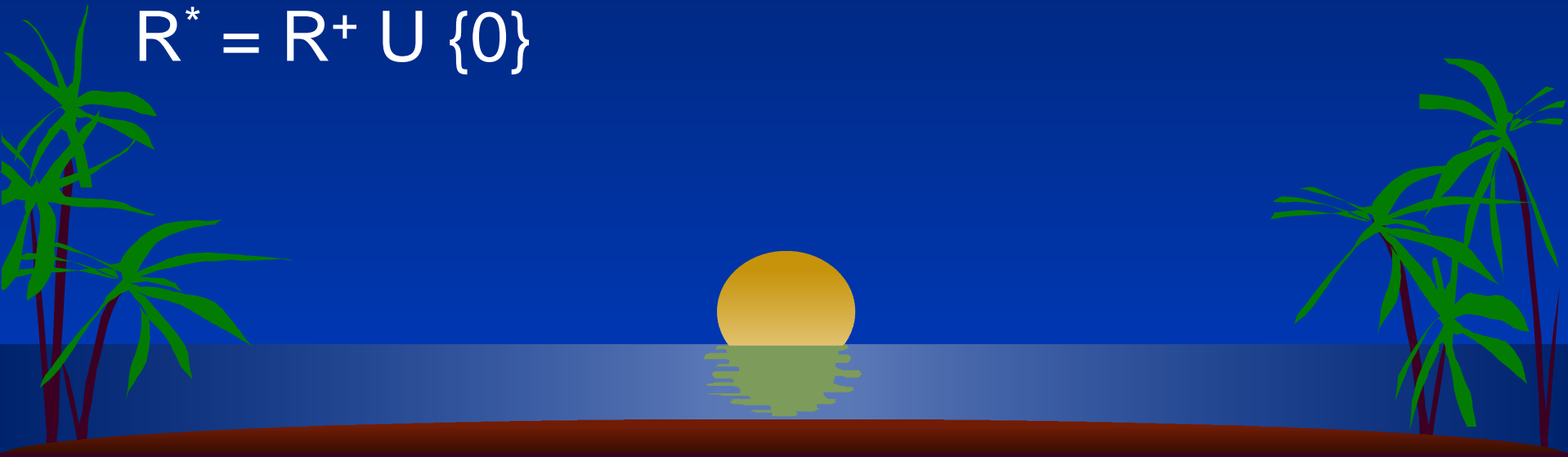
$$\mathbb{N} = \{0, 1, 2, 3, \dots\}$$

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$\mathbb{R}$  = Set of Reals

$\mathbb{R}^+$  = Set of Positive Reals

$$\mathbb{R}^* = \mathbb{R}^+ \cup \{0\}$$



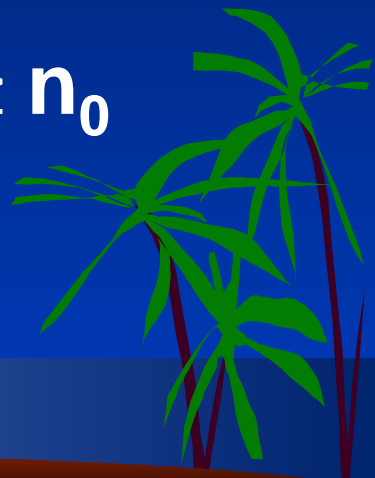
# Comparing $f(n)$ and $g(n)$

Let  $f$  be a function from  $\mathbf{N}$  to  $\mathbf{R}$ .

$\mathbf{O}(f)$  (Big  $\mathbf{O}$  of  $f$ ) is the set of all functions  $g$  from  $\mathbf{N}$  to  $\mathbf{R}$  such that:

1. There exists a real number  $c > 0$
2. AND there exists an  $n_0$  in  $\mathbf{N}$

Such that:  $g(n) \leq cf(n)$  whenever  $n \geq n_0$



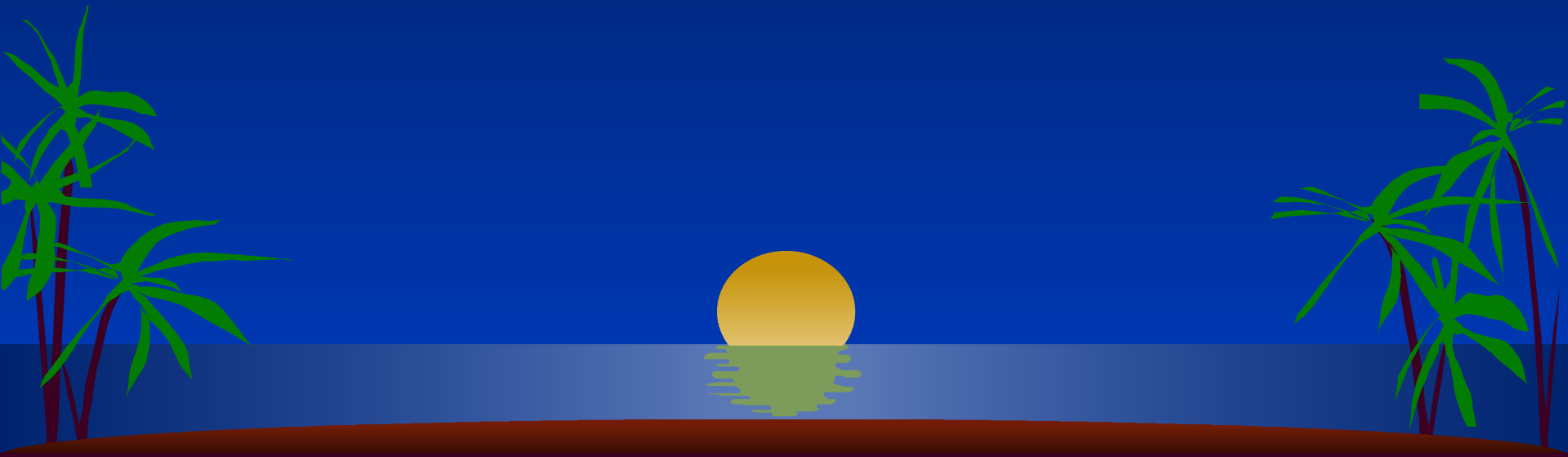
# Notation and Pronunciation

Proper Notation:  $g \in O(f)$

“g is oh of f”

Also Seen:

$$g = O(f)$$



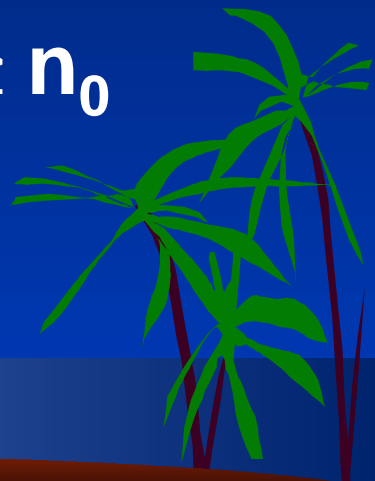
# Big Omega

Let  $f$  be a function from  $\mathbf{N}$  to  $\mathbf{R}$ .

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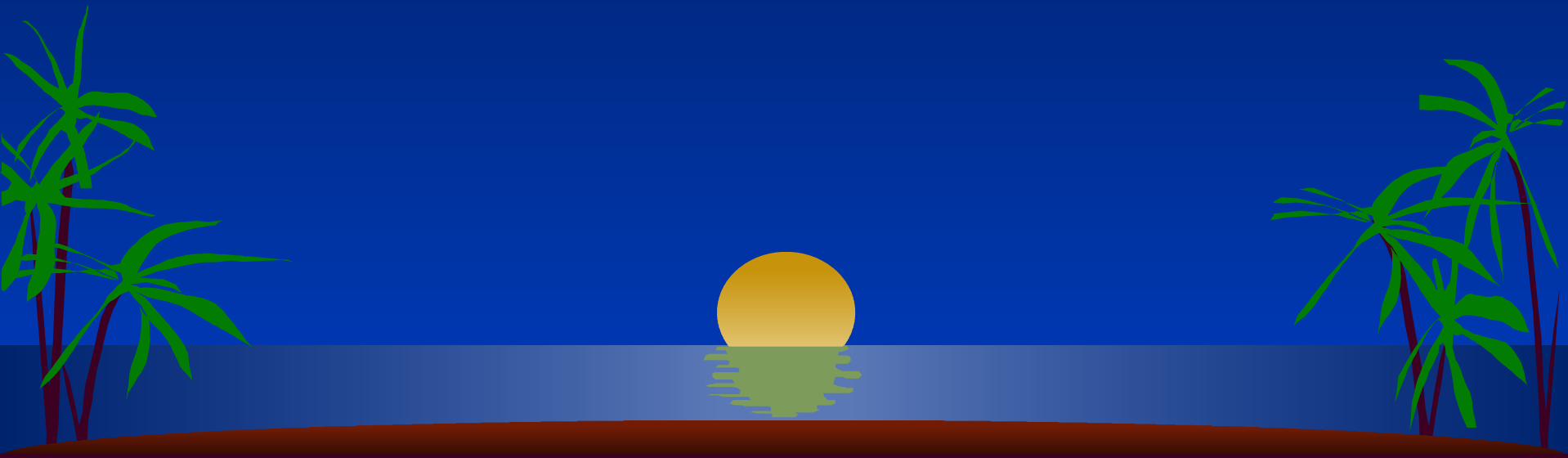
# Big Theta

$$\Theta(f) = O(f) \cap \Omega(f)$$

“g is of Order f”

$$g \in \Theta(f)$$

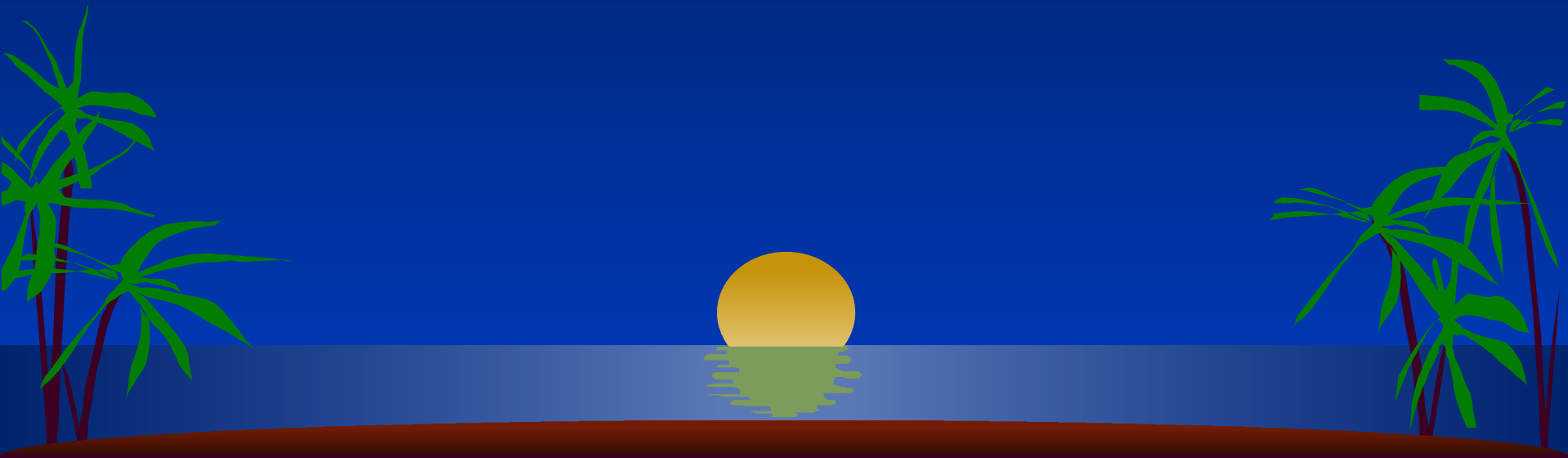
“g is Order f”



# Little o and Little Omega

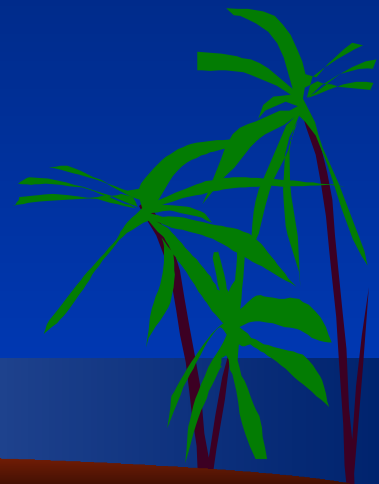
$$o(f) = O(f) - \Theta(f)$$

$$\omega(f) = \Omega(f) - \Theta(f)$$



# English Interpretations

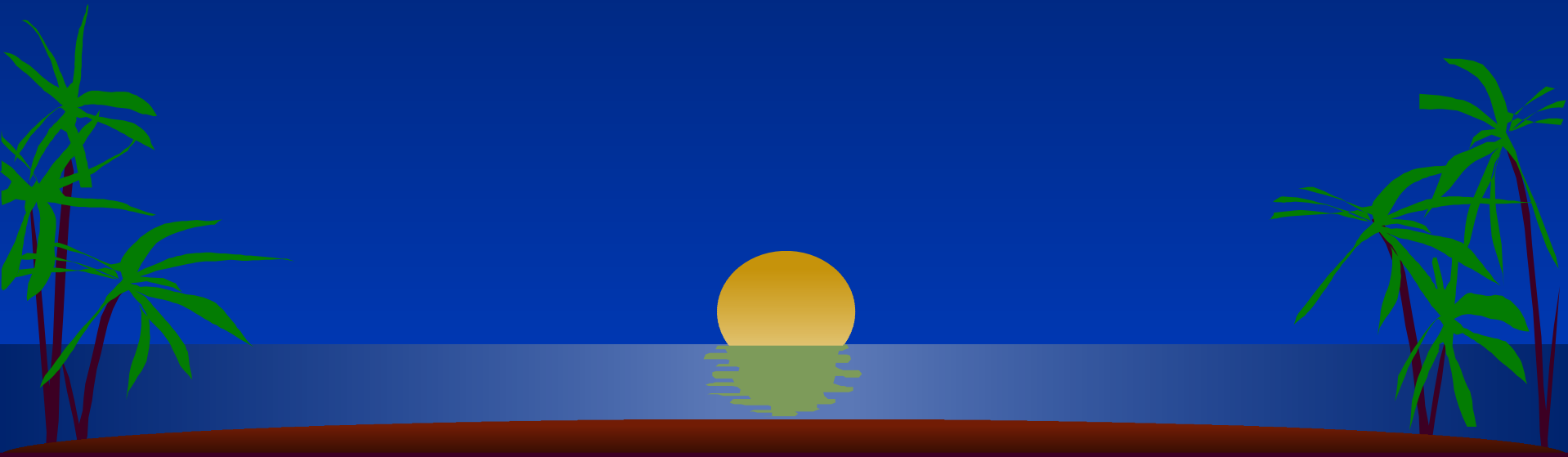
- $O(f)$  - Functions that grow no faster than  $f$
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- $\Theta(f)$  - Functions that grow at the same rate as  $f$
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# Limit Formulas

$$g \in O(f) \quad \underline{\text{if}} \quad \lim_{n \rightarrow \infty} \frac{g(n)}{f(n)} = c, \quad \underline{\text{for some}} \quad c \in R^*$$

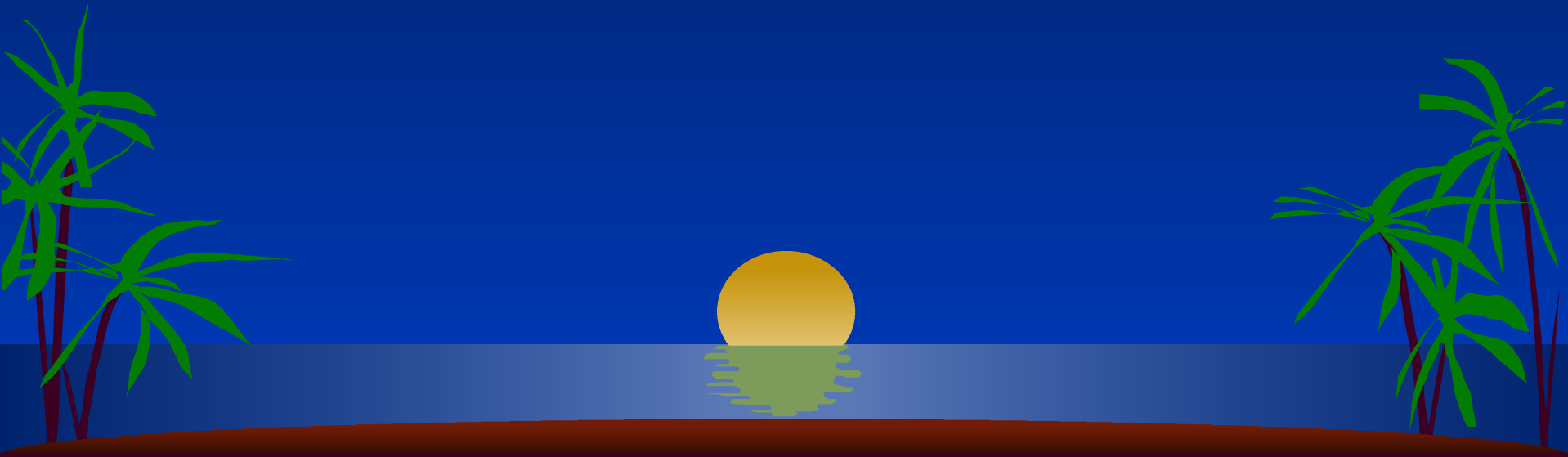
$$g \in \Omega(f) \quad \underline{\text{if}} \quad \lim_{n \rightarrow \infty} \frac{g(n)}{f(n)} = \infty \quad \underline{\text{or}} \quad \lim_{n \rightarrow \infty} \frac{g(n)}{f(n)} = c > 0$$



# More Limit Formulas

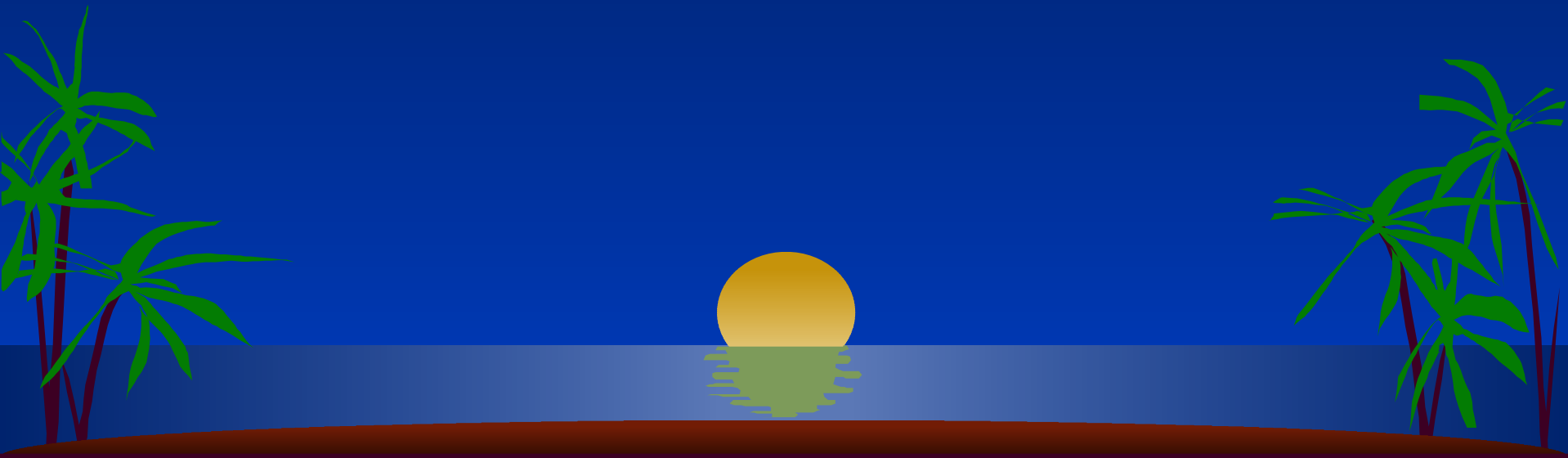
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$$g \in o(f) \quad \underline{\text{if}} \quad \lim_{n \rightarrow \infty} \frac{g(n)}{f(n)} = 0$$



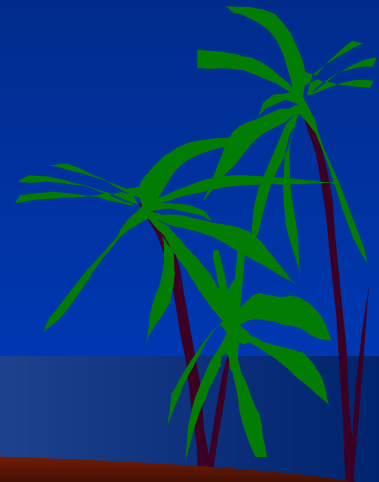
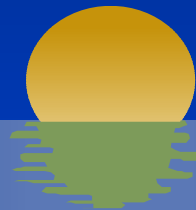
# The Last Limit Formula

$$g \in W(f) \quad \underline{\text{if}} \quad \lim_{n \rightarrow \infty} \frac{g(n)}{f(n)} = \infty$$



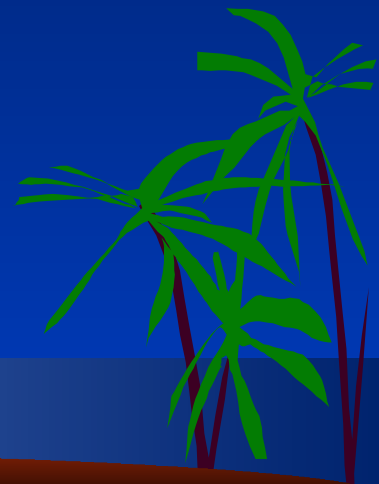
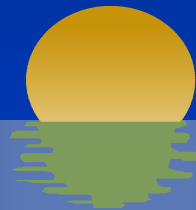
# Properties

- Transitivity
  - if  $f \in O(g)$  and  $g \in O(h)$  then  $f \in O(h)$
  - Same holds for  $\Theta$ ,  $\Omega$ ,  $o$ , and  $\omega$
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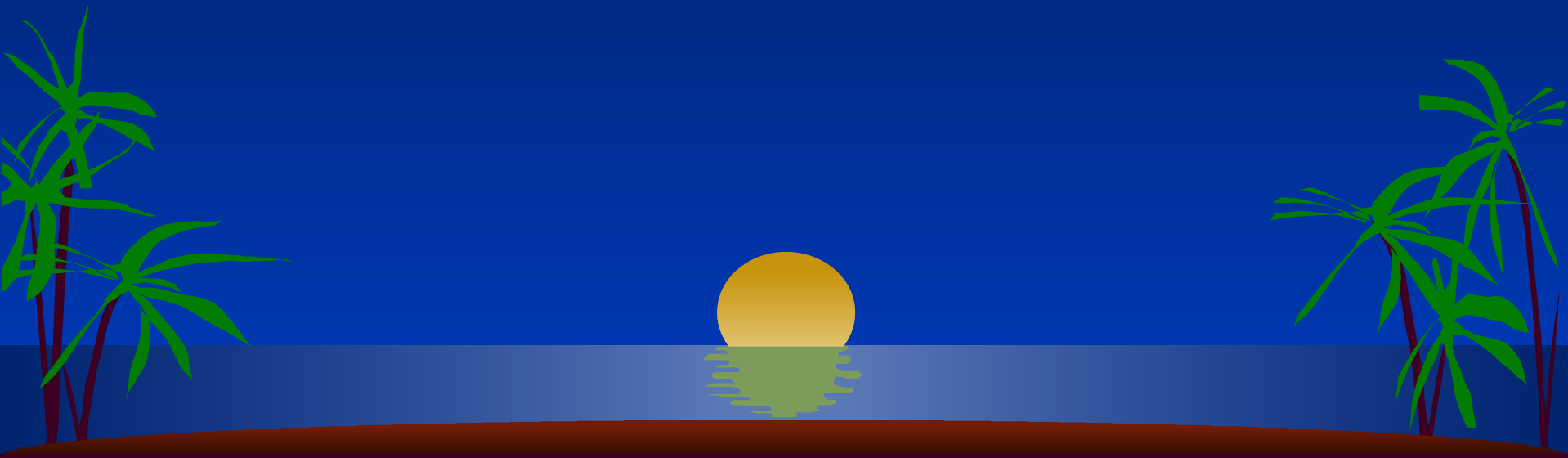
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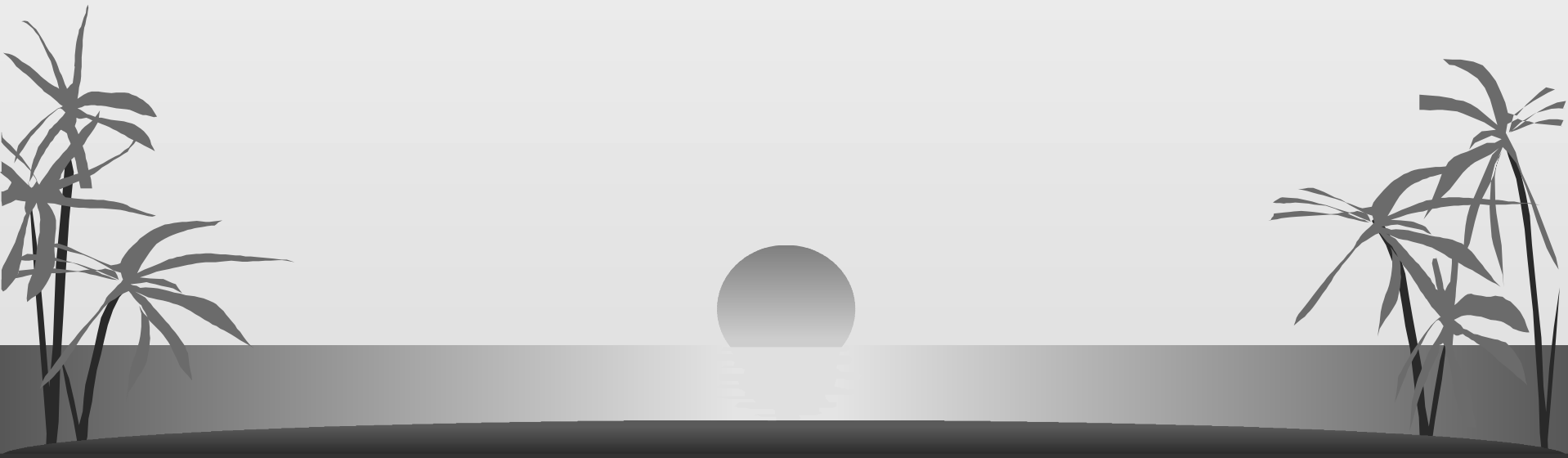


# And Even More Properties

- Big Theta is an equivalence relation
  - $f \sim g$  if and only if  $f \in \Theta(g)$
- $O(f+g) = O(\max(f,g))$ 
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# Comparing Functions



# Notes on Notation

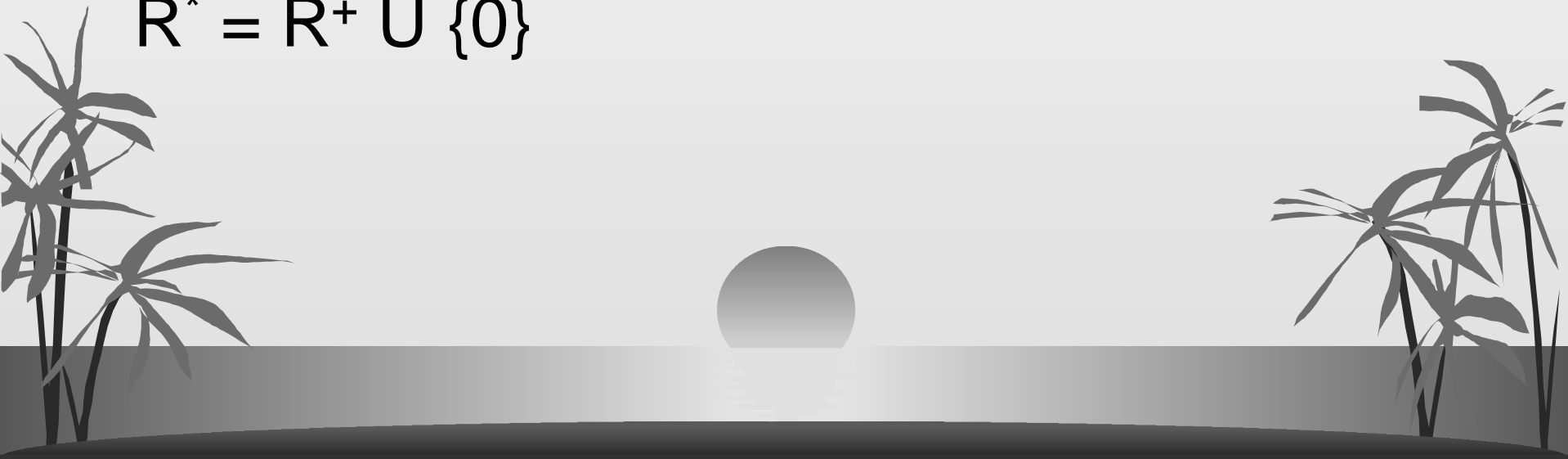
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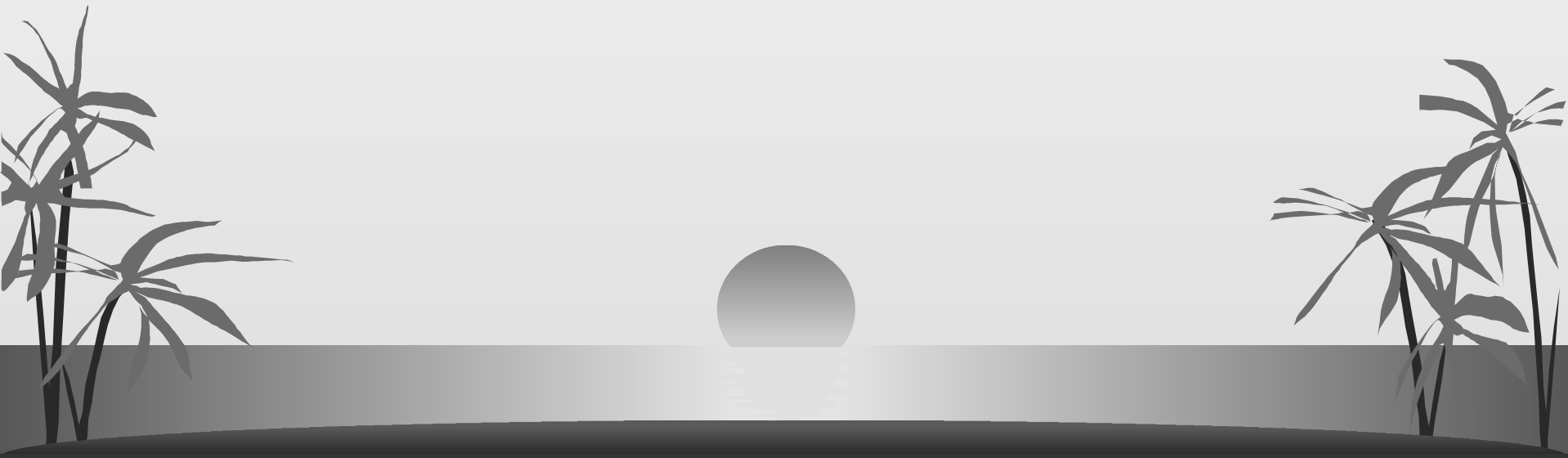


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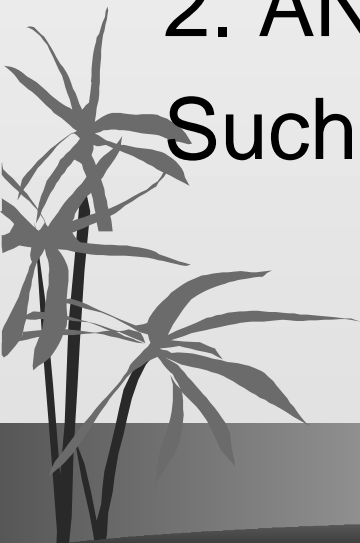
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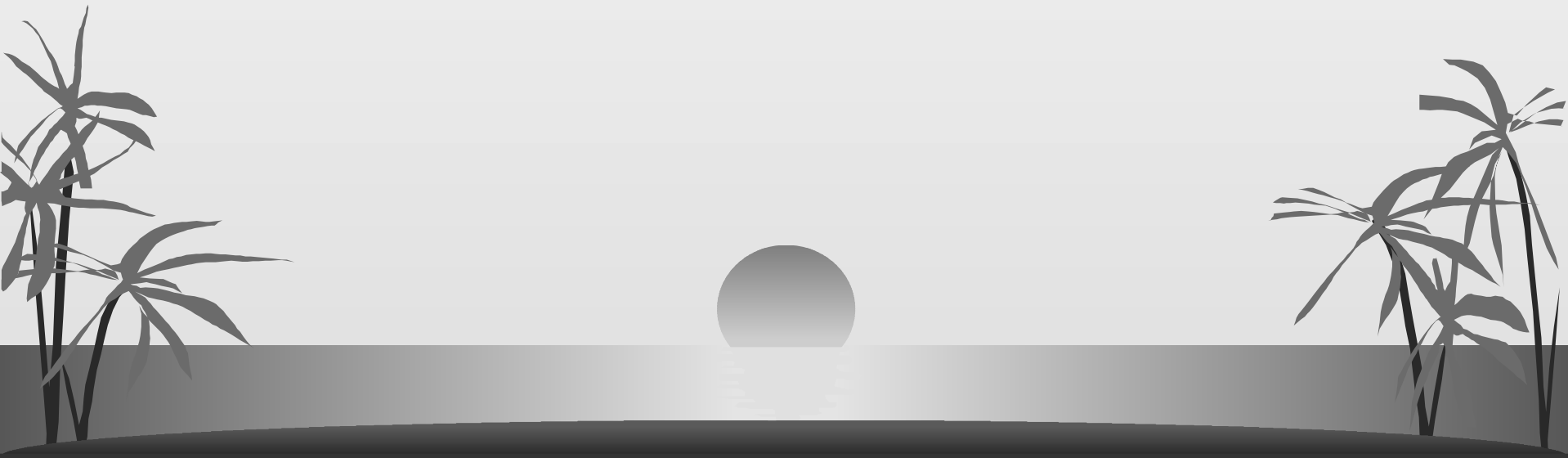
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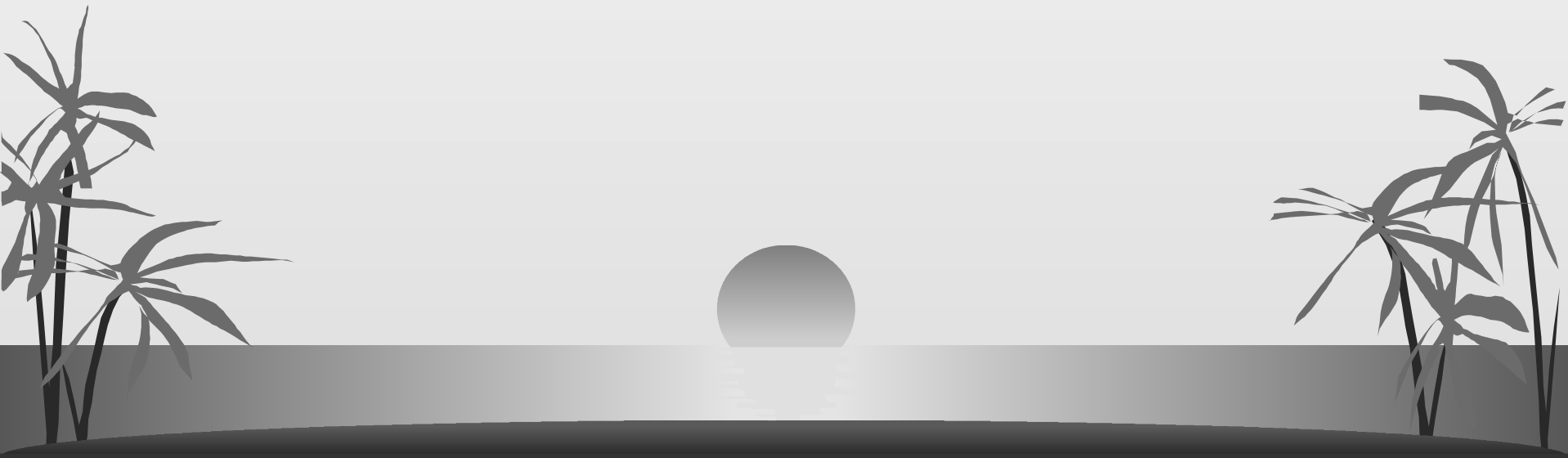
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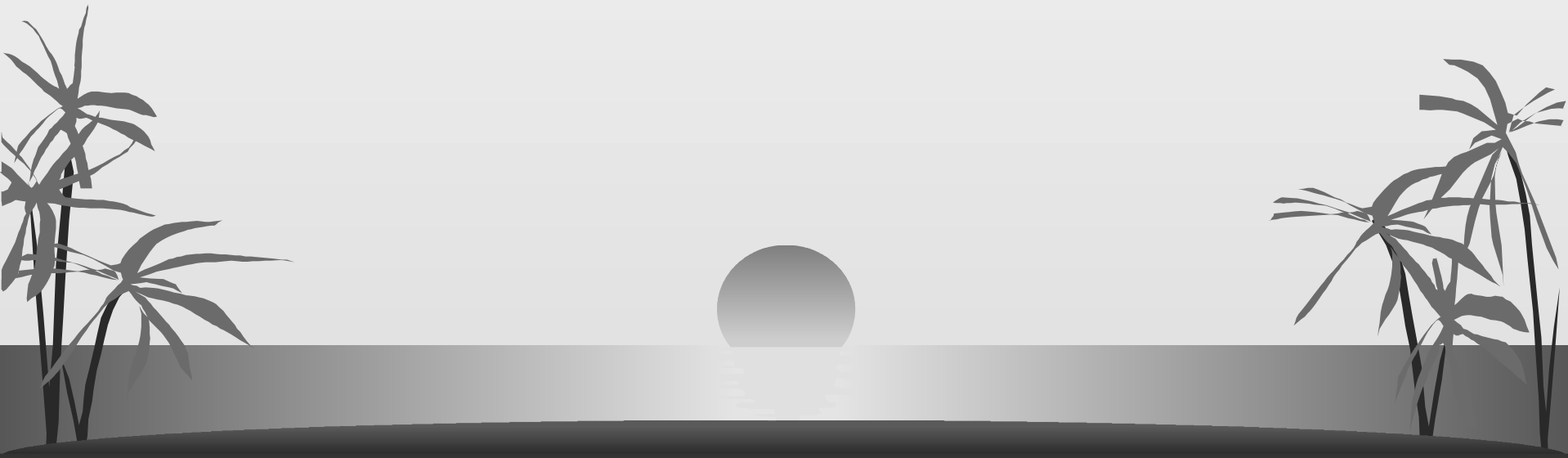
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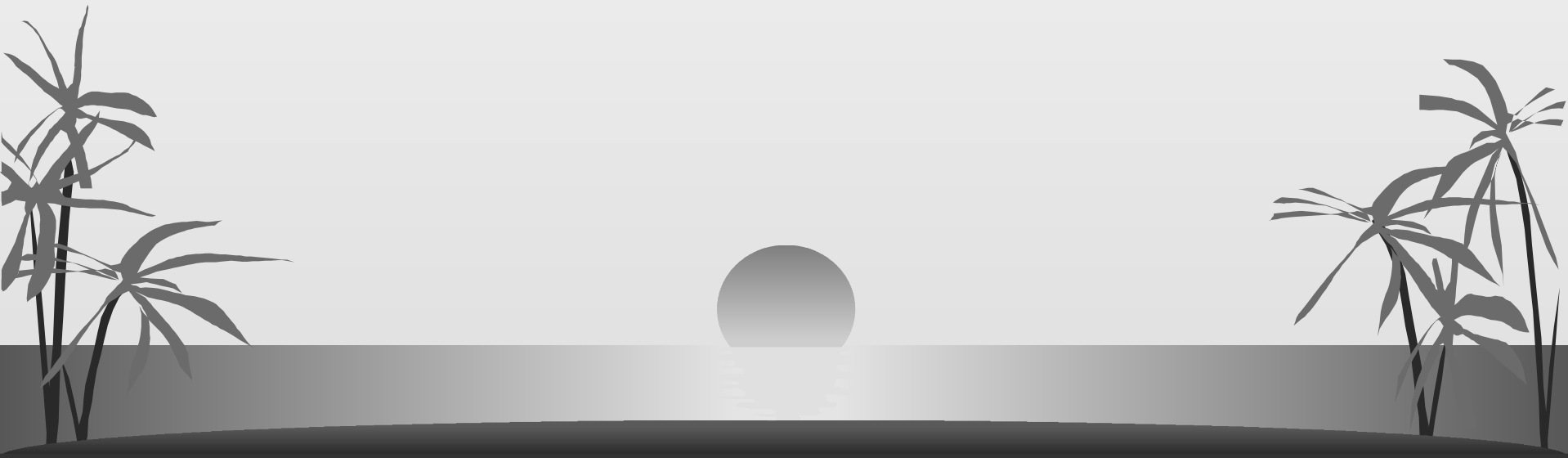
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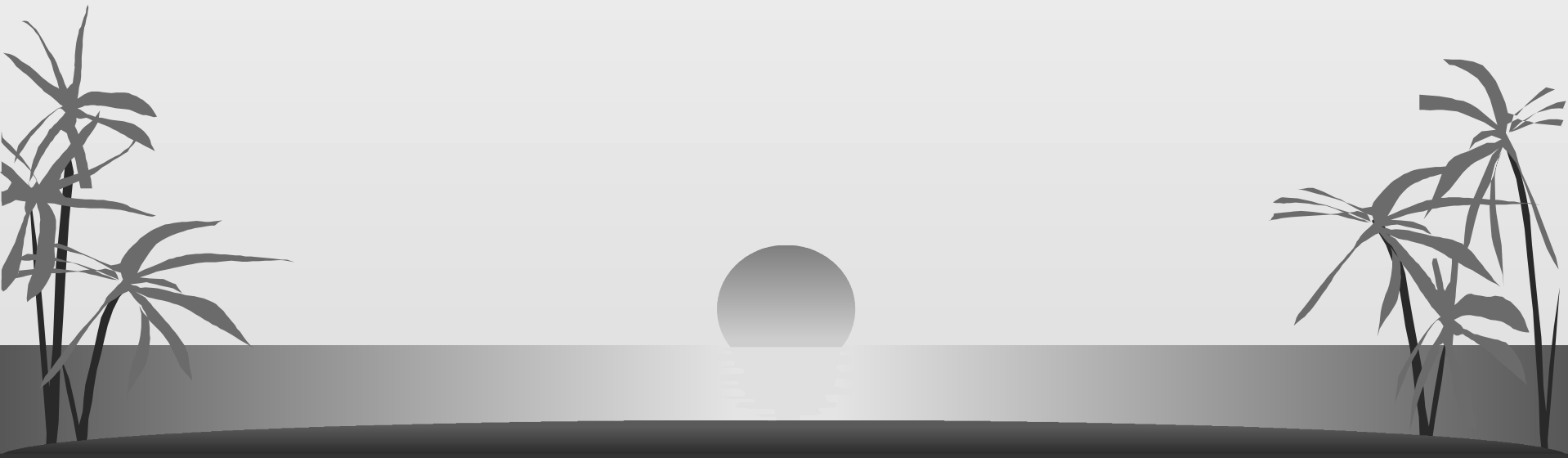
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# The Last Limit Formula

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