SOLUTION TO PROBLEM 3.1-3

by

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\[ \sum_{k=0}^{\infty} \frac{k-1}{2^k} = -1 + 0 + 1 + \frac{2}{4} + \frac{3}{8} + \ldots \]

Let \( \frac{1}{2} = x \), then this equation can be rewritten:

\[ \sum_{k=0}^{\infty} (k-1)x^k = -1 + 0 + x^2 + 2x^3 + 3x^4 + \ldots \]

This series is related to the series \( \sum_{k=0}^{\infty} x^k \) in the following way.

In the radius of convergence, the derivative of a power series is obtained by taking the derivative of each term. Thus:

\[ \frac{d}{dy} \sum_{k=0}^{\infty} x^k = \sum_{k=0}^{\infty} kx^{k-1} = 0 + 1x + 2x^2 + 3x^3 + \ldots \]

And:

\[ \sum_{k=0}^{\infty} (k-1)x^k = -1 + x^2 \sum_{k=0}^{\infty} kx^{k-1} \]

Now returning to \( \sum_{k=0}^{\infty} x^k \), the following equation can be found in the book:

\[ \sum_{k=0}^{\infty} x^k = \frac{1}{1-x} \]

The derivative of \( \frac{1}{1-x} \) is \( \frac{1}{(1-x)^2} \), so

\[ \sum_{k=0}^{\infty} (k-1)x^k = -1 + \frac{x^2}{(1-x)^2} \]

Substituting \( 1/2 \) back in for \( x \), we get

\[ \sum_{k=0}^{\infty} \frac{(k-1)}{2^k} = -1 + \frac{\left(\frac{1}{2}\right)^2}{\left(1 - \frac{1}{2}\right)^2} = -1 + \frac{\left(\frac{1}{2}\right)^2}{\left(\frac{1}{2}\right)^2} = -1 + 1 = 0 \]