1. Let $S$ be a set of ordered pairs such that
   - Initial - $(0, 0) \in S$
   - Step - $(a, b) \in S \rightarrow (a + 2, b + 3) \in S \land (a + 3, b + 2) \in S$

   (a) Show $S$ after 4 applications of the step.
   (b) Use strong induction on the number of applications of the step that $(a, b) \in S \rightarrow 5|(a+b)$, where $5|(a+b)$ means $\frac{a+b}{5}$ has a remainder of 0.

2. Assume that a chocolate bar consists of $n$ squares arranged in a rectangular pattern. The entire bar, or any smaller rectangular piece of the bar, can be broken along a vertical or horizontal line separating the squares. Assuming that only one piece can be broken at a time, use strong induction to show that $n - 1$ breaks are needed to create $n$ separate pieces.

3. From the following functional definition, find a formula for $f(n)$ and prove your formula correct.
   
<table>
<thead>
<tr>
<th>$f(0)$</th>
<th>$f(1)$</th>
<th>$\forall n \geq 2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>$f(n) = 2 \times f(n-1)$</td>
</tr>
</tbody>
</table>

4. Given $A$ and $A^n$ below. Prove that the matrix $A$ raised to the $n$th power is $A^n$, where $f_n$ is the $n$th Fibonacci number.

5. Give a recursive algorithm with input $n$, an integer, for finding the sum of the squares of the first $n$ integers

$$A = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} \quad A^n = \begin{bmatrix} f_{n+1} & f_n \\ f_n & f_{n-1} \end{bmatrix}$$