1. Prove that any integer \( n \) greater than or equal to 1 can be represented as \( n = 2^i \ast q \) where \( i \geq 0 \) and \( q \) is odd.

2. Let \( d \) be a positive integer. Show that among any group of \( d + 1 \) integers, there are two with exactly the same remainder when divided by \( d \). Note that the integers do not have to be consecutive.

3. A particular brand of shirt comes in 12 colors, has a male and female version, and has 3 sizes for each gender. How many different types of shirts are there?

   (a) How many are there?
   (b) How many begin and end with 1?
   (c) How many begin and end with the same two bits (in order)?

5. Consider positive integers between 1000 and 9999 (inclusive)
   (a) Are divisible by 9?
   (b) are even?
   (c) have distinct digits?
   (d) are not divisible by 3?
   (e) are divisible by 5 or 7?
   (f) are not divisible by either 5 or 7? (but not both)
   (g) are divisible by 5 but not by 7?
   (h) are divisible by 5 and 7?