1. Let $G = (V_1 \cup V_2, E)$ be a weighted complete bipartite graph where $V_1 = \{1, 3, 5\}, V_2 = \{2, 4, 6\}$ and $\forall e \in E, e = (x, y, x + y)$ (i.e., the weight of an edge is the sum of the two vertices incident on the edge).

   (a) Draw the graph.
   (b) Produce the matrix corresponding to the graph
   (c) Use Dijkstra's Algorithm to find the shortest path from 6 to 2. Show the table described in the notes before the first iteration and after each iteration

2. Prove or disprove that in a weighted complete bipartite graph, the “shortest path” can never contain more than two edges.

3. Prove that the relation $R$ corresponding to the DAG $G$ must be antisymmetric.

4. Consider the matrix $M$ below.

   (a) Prove the graph $G$ corresponding to $M$ is a tree
   (b) Show the rooted tree $T$ with 1 as the root.
   (c) What is the height and the branching factor of $T$?
   (d) Show the rooted tree $T'$ with 4 as the root.
   (e) What is the height and the branching factor of $T'$?

\[
M = \begin{bmatrix}
0 & 1 & 0 & 0 & 1 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 1 \\
1 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 1 & 1 & 1 & 0
\end{bmatrix}
\]