1. Let $A = \{0, 1, 2, 3\}$, $f : A \rightarrow A$.

(a) If $f(a) = (a+1) \% 4$, is $f$ 1-1? if $f$ onto? is $f$ a function?
- 1-1, onto and a function

(b) If $f(a) = (a+1) \% 4$, is $f^{-1}$ 1-1? if $f^{-1}$ onto? is $f^{-1}$ a function?
- 1-1, onto and a function

(c) If $f(a) = (2a) \% 4$, is $f$ 1-1? if $f$ onto? is $f$ a function?
- Not 1-1, not onto and a function

(d) If $f(a) = (2a) \% 4$, is $f^{-1}$ 1-1? if $f^{-1}$ onto? is $f^{-1}$ a function?
- Not 1-1, Not onto and not a function

2. Let $f : A \rightarrow A$ such that $f$ is a bijection and $R(A, A)$ such that $R$ is symmetric. Prove or disprove that $f \subset R \rightarrow f^{-1} \subset R$.

- Since $f$ is a bijection every element of the co-domain $A$ has exactly one domain element mapped to it. Also, $R(A, A)$ is symmetric so for every element $(a, b)$ in $R$ there is also an element $(b, a)$ in $R$.

- If $f \subset R$, every ordered pair $(a, b)$ of $f$ also belongs to $R$. $f^{-1}$ contains elements of $f$ in reversed form. That means for every $(a, b) \in R$, $(b, a) \in R^{-1}$. Since $R$ is symmetric, it contains all such $(b, a)$ ordered pairs. So $f^{-1} \subset R$.

Proved

3. Let $A$ and $B$ be arbitrary sets. Let $R(A, B)$. Provide a predicate for $R$ such that $R$ would meet the definition of a 1-1 function.

For $R$ to be a 1-1 function, it must be a function and it must be 1-1.

Let $P$ be the predicate for a function $\forall (a, b), (a_1, b_1) \in R, a = a_1 \rightarrow b = b_1$
Then a 1-1 function can be defined as \( P \land \forall (a, b), (a_1, b_1) \in R, b = b_1 \rightarrow a = a_1. \)

4. Let A and B be arbitrary sets. Let \( R(A, B) \). Provide a predicate for \( R \) such that \( R \) would meet the definition of an onto function.

For \( R \) to be a onto function, it must be a function and it must be onto.

Let \( P \) be the predicate for a function \( \forall (a, b), (a_1, b_1) \in R, a = a_1 \rightarrow b = b_1 \)

Then an onto function can be defined as \( P \land \forall b \in B, a \in A \) such that \( (a, b) \in R \).

5. Let \( f: \mathbb{Z} \rightarrow \mathbb{Z} \) and \( g: \mathbb{Z} \rightarrow \mathbb{Z} \). For each pair of functions, produce a formula for \( f(g(x)) \) and \( g(f(x)) \).

(a) \( f(x) = 2^x \), \( g(x) = x + 1 \)

\( f(g(x)) = 2^{x+1} \)
\( g(f(x)) = 2^x + 1 \)

(b) \( f(x) = \lceil x/2 \rceil \), \( g(x) = 2x \)

\( f(g(x)) = \lceil 2x/2 \rceil = x \)
\( g(f(x)) = 2\lceil x/2 \rceil \)

(c) \( f(x) = 2x \), \( g(x) = x + 1 \)

\( f(g(x)) = 2x+2 \)
\( g(f(x)) = 2x+1 \)