1. Compute the following, or indicate if it cannot be computed.

a) \( M \times N \)
\[
\begin{bmatrix}
3 & 3 & 3 \\
-3 & -3 & -3
\end{bmatrix}
\]

b) \( M \times O \)
- Cannot be computed

c) \( O \times M \)
\[
\begin{bmatrix}
2 & -2 \\
-2 & 2 \\
1 & -1
\end{bmatrix}
\]

d) \( N \times O \)
\[
\begin{bmatrix}
1 & 1 \\
1 & 4
\end{bmatrix}
\]

e) \( O \times N + M \)
- Cannot be computed

f) \((O \times N + M)^2\)
- Cannot be computed

g) \( M^3 \)
\[
\begin{bmatrix}
-4 & 4 \\
4 & -4
\end{bmatrix}
\]

2. Let \( A = \{1, 2, 3, 4, 5, 6\} \). For each \( R(A, A) \), create the corresponding matrix \( MR \). Order the elements in \( A \) by ascending value.

a is represented as rows and b as columns in the following matrices.
(a) $R=\{(a,b)|a \leq b\}$

$$
\begin{bmatrix}
1 & 1 & 1 & 1 & 1 & 1 \\
0 & 1 & 1 & 1 & 1 & 1 \\
0 & 0 & 1 & 1 & 1 & 1 \\
0 & 0 & 0 & 1 & 1 & 1 \\
0 & 0 & 0 & 0 & 1 & 1 \\
0 & 0 & 0 & 0 & 0 & 1 \\
\end{bmatrix}
$$

(b) $R=\{(a,b)|a \% 2 = 0 \land b \% 2 = 1\}$

$$
\begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 1 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 1 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 1 & 0 & 1 & 0 \\
\end{bmatrix}
$$

(c) $R=\{(a,b)|a + b = 7\}$

$$
\begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
$$

3. Consider the identity matrix $I$ as a boolean matrix. Consider an arbitrary relation $R(A, A)$ such that $M_R=I$. Using $I$, prove or disprove $R$ is reflexive, symmetric, antisymmetric and transitive.

A 3*3 identity matrix is given below:

$$
\begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1 \\
\end{bmatrix}
$$

All the diagonal elements are 1. So for a relation $R(A,A)$ such that $M_R=I$, all the diagonal elements in $M_R$ are 1. So $R$ is reflexive.

In case of identity matrix for all $i$ and $j$, $m_{ii} = m_{ij}$. For a relation $R(A,A)$ such that $M_R=I$, for all $i$ and $j$ in $M_R$, $m_{ji} = m_{ij}$. So $R$ is symmetric.

For $I$, for all $i$ and $j$ $m_{ji} = m_{ij}=1$ implies $i=j$. All the non diagonal elements are 0. So whenever $m_{ij}=m_{ji}=1$ it means $i=j$. So for a relation $R(A,A)$ such that $M_R=I$, for all $i$ and $j$ in $M_R$, whenever $m_{ij}=m_{ji}=1$ it means $i=j$. So $R$ is antisymmetric.
For a relation $R(A,A)$ such that $M_R=I$, for all $i,j$ and $k$, whenever $m_{ij}=1$ and $m_{jk}=1$, $m_{ik}$ is also equal to 1. So $R$ is transitive.

4. Consider the matrix $P$ below. Consider an arbitrary relation $R(A, A)$ such that $M_R=P$. Using $P$, prove or disprove $R$ is reflexive, symmetric, antisymmetric and transitive.

Matrix $P$ is given below:

$$
egin{bmatrix}
1 & 0 & 1 \\
0 & 1 & 0 \\
1 & 0 & 1
\end{bmatrix}
$$

All the diagonal elements are 1. So for a relation $R(A,A)$ such that $M_R=P$, all the diagonal elements in $M_R$ are 1. So $R$ is reflexive.

For matrix $P$, $m_{ji}=m_{ij}$ for all $i$ and $j$. For a relation $R(A,A)$ such that $M_R=P$, for all $i$ and $j$ in $M_R$, $m_{ji}=m_{ij}$ So $R$ is symmetric.

For Relation $R$ to be antisymmetric, the matrix $M_R$ satisfy: $m_{ji}=m_{ij}=1$ implies $i=j$ for all $i$ and $j$. But in matrix $P$, $m_{31}=m_{13}=1$ and $i \neq j$ in that case. So $R$ is not antisymmetric.

For a relation $R(A,A)$ such that $M_R=I$, for all $i,j$ and $k$, whenever $m_{ij}=1$ and $m_{jk}=1$, $m_{ik}$ is also equal to 1. So $R$ is transitive.

5. Consider the matrix answer to 2(c). Use this matrix to prove or disprove $R$ is reflexive, symmetric, antisymmetric and transitive.

Matrix answer to 2(c) is given below:

$$
egin{bmatrix}
0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
$$

The diagonal elements are 0. The condition required for reflexive property is $m_{ii}=1$ for all $i$. But all $m_{ii}$ is 0 in the above matrix. So $R$ is not reflexive.

For the above matrix, $m_{ji}=m_{ij}$ for all $i$ and $j$. So $R$ is symmetric.
For Relation R to be antisymmetric, the matrix $M_R$ satisfy: $m_{ij} = m_{ji}=1$ implies $i=j$ for all $i$ and $j$. But in the above matrix, $m_{61} = m_{16}=1$ and $i \neq j$ in that case.
So $R$ is not antisymmetric.

For a relation $R(A,A)$ to be transitive the following condition must hold: In matrix $M_R$ Whenever $m_{ij}=1$ and $m_{jk}= 1$, $m_{ik}$ is also equal to 1, for all $i,j$ and $k$.
In the above matrix $m_{16}=1$ and $m_{61}=1$ but $m_{11}=0$. So this violates the transitivity condition.
Hence, $R$ is not transitive.