1. Prove that \( \sum_{k=0}^{n} k * k! = (n + 1)! - 1 \).

Using Mathematical Induction to prove the statement.
Let \( P(n) \) be the statement that \( \sum_{k=0}^{n} k * k! = (n + 1)! - 1 \).

Basis Step (\( P(0) \)):
LHS: \( \sum_{k=0}^{0} k * k! = 0 * 0! = 0 \)
RHS: \( (0+1)! - 1 = 1! - 1 = 0 \)
\( P(0) \) is true.

Inductive Step:
Assume \( P(m) \) is true. So \( \sum_{k=0}^{m} k * k! = (m + 1)! - 1 \).

Now we have to prove \( P(m+1) \) is true.
We have to prove \( \sum_{k=0}^{m+1} k * k! = (m + 2)! - 1 \).

LHS: \( \sum_{k=0}^{m+1} k * k! = \sum_{k=0}^{m} k * k! + (m+1) * (m+1)! \)
\( = (m+1)! - 1 + (m+1) * (m+1)! \)
\( = (m+1)! + (m+1)! - 1 \)
\( = (m+2)! - 1 \)
\( = \text{RHS} \)

Hence we have proven that if \( P(m) \) is true \( P(m+1) \) is true.
The given statement is proved.

2. Prove that \( \sum_{k=1}^{n} \frac{1}{k*(k+1)} = \frac{n}{n+1} \).

Using Mathematical Induction to prove the statement.
Let \( P(n) \) be the statement that \( \sum_{k=1}^{n} \frac{1}{k*(k+1)} = \frac{n}{n+1} \).

Basis Step (\( P(1) \)):
LHS: \( \sum_{k=1}^{1} \frac{1}{k*(k+1)} = 1/(1*2) = 1/2 \)
RHS: \( 1/(1+2) = 1/2 \)
P(0) is true.

Inductive Step:
Assume P(m) is true. So \[ \sum_{k=1}^{m} \frac{1}{k(k+1)} = \frac{m}{m+1}. \]

Now we have to prove P(m+1) is true.
We have to prove \[ \sum_{k=1}^{m+1} \frac{1}{k(k+1)} = \frac{m+1}{m+2}. \]

LHS:
\[ \sum_{k=1}^{m+1} \frac{1}{k(k+1)} = \sum_{k=1}^{m} \frac{1}{k(k+1)} + \frac{1}{(m+1)((m+1)+1)} \]
\[ = \sum_{k=1}^{m} \frac{1}{k(k+1)} + \frac{1}{(m+1)(m+2)} \]
\[ = \frac{m}{m+1} + \frac{1}{(m+1)(m+2)} \]
\[ = \frac{m(m+2)+1}{(m+1)(m+2)} \]
\[ = \frac{m+1}{m+2} \]
Hence we have proven that if P(m) is true P(m+1) is true.
The given statement is proved.

3. Prove that \( n > 6 \rightarrow 3^n < n! \)

Using Mathematical Induction to prove the statement.
Let P(n) be the statement that \( n > 6 \rightarrow 3^n < n! \)

Basis Step (P(7)):
For n=7, 7>6 and \( 3^7 = 2187 < 7!(= 5040) \) P(7) is true.

Inductive Step:
Assume P(m) is true where \( m > 6 \). So \( 3^m < m! \)

Now we have to prove P(m+1) is true.
m+1 is also greater than 6.
\[ 3^{m+1} \]
\[ = 3^m * 3 \]
We know that \( 3^m < m! \) and \( 3 < m + 1 \). So
\[ 3^m * 3 < m! * (m + 1) \]
So \( 3^{m+1} < m! * (m + 1) \) which means \( 3^{m+1} < (m + 1)! \)
m + 1 > 6. So \( 3^{m+1} < (m + 1)! \)

Hence we have proven that if P(m) is true P(m+1) is true.
The given statement is proved.

4. Prove that if \( A_0, A_1, A_2...A_{n-1} \) and B are sets then \( \cap A_i \cup B = (A_0 \cup B) \cap (A_1 \cup B) \cap .... \cap (A_{n-1} \cup B) \).
Using Mathematical Induction to prove the statement. Let $P(n)$ be the statement that if $A_0, A_1, A_2, ..., A_{n-1}$ and $B$ are sets then $\bigcap \ A_i \cup B = (A_0 \cup B) \cap (A_1 \cup B) \cap ... \cap (A_{n-1} \cup B)$.

Basis Step ($P(2)$):
$A_0, A_1$ and $B$ are sets
LHS: $\bigcap \ A_i \cup B$
$= (A_0 \cap A_1) \cup B$
$= (A_0 \cup B) \cap (A_1 \cup B)$ (By using Distributive law of sets)
=RHS
$P(2)$ is proved

Inductive Step:
Assume $P(m)$ is true. So $A_0, A_1, A_2, ..., A_{m-1}$ and $B$ are sets and $\bigcap A_i \cup B = (A_0 \cup B) \cap (A_1 \cup B) \cap ... \cap (A_{m-1} \cup B)$.

Now we have to prove $P(m+1)$ is true.
Let $Y$ be the set $A_0 \cap A_1 \cap ... \cap A_{m-1}$
\[ \bigcap A_i \cup B \]
$= (A_0 \cap A_1 \cap ... \cap A_{m-1} \cap A_m) \cup B$
$= (Y \cap A_m) \cup B$
$= (Y \cup B) \cap (A_m \cup B)$
$= ((A_0 \cap A_1 \cap ... \cap A_{m-1}) \cup B) \cap (A_m \cup B)$
$= (A_0 \cup B) \cap (A_1 \cup B) \cap ... \cap (A_{m-1} \cup B) \cap (A_m \cup B)$

So $P(m+1)$ is true.
Hence we have proven that if $P(m)$ is true $P(m+1)$ is true.
The given statement is proved.