Discrete Structure-Assignment 14

March 2019

1. How many strings of 4 uppercase English letters are there?
   (a) if letters can be repeated?
   \[ 26^4 = 456976 \]
   (b) that start with X, if no letters can be repeated?
   \[ 25 \times 24 \times 23 = 13800 \]
   (c) that start with the letters BU, if letters can be repeated?
   \[ 26 \times 26 = 676 \]
   (d) that start with the letters BU, if no letters can be repeated?
   \[ 24 \times 23 = 552 \]

2. How many different sets of three positive integers less than 100 be chosen?
   \[ \binom{99}{3} = \frac{99!}{3! \times (99-3)!} = 156849 \]

3. Four cards are randomly drawn (and replaced in the deck) from a deck of 13 cards (all hearts). How many possible outcomes
   (a) are there in total?
   \[ 13^4 = 28561 \]
   (b) contain exactly 2 face cards?
   - Since two cards are face cards, there are two cards which can take any non face cards. So the total number of combinations for two non face cards and two face cards= \[ 3 \times 3 \times 10 \times 10 = 900 \]
   But 2 face cards can be picked in any two of the possible 4 picks. Number of ways two face cards can be picked = \( \binom{4}{2} \) = \( \frac{4!}{2! \times 2!} \) = 6
   So total number of ways to pick 2 face cards in 4 draws = 900 \times 6 = 5400
   (c) contain at least 2 face cards?
   Number of outcomes which contain at least 2 face cards = Number of outcomes which contain 2 face cards + Number of outcomes which contain 3 face cards + Number of outcomes which contain 4 face cards

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Number of outcomes which contain 2 face cards = 5400 (from number (b))
Number of outcomes which contain 3 face cards = \( C(4,3) \times (3 \times 3 \times 3 \times 10) = 4 \times 270 = 1080 \) (Similar to (b))
Number of outcomes which contain 4 face cards = \( C(4,4) \times (3 \times 3 \times 3 \times 3) = 81 \)
Number of outcomes which contain at least 2 face cards = 5400 + 1080 + 81 = 6561

(d) contain the same number of face cards and non-face cards?
- 5400 (Same case as (b))

What is the coefficient of \( x^5 \) in \((1 + 2x)^{10}\)?
- Coefficient of \( x^5 \) in \((1 + 2x)^{10}\) = \( C(10,5) \times 1^5 \times 2^5 = 8064 \)