1. Consider hands of size 1, 5, 10 and 13 and a deck of 52 cards. You should have 4 answers for each question. You can leave combination or permutation formulas in the answer.

(a) How many different hands are possible?
Number of possible hands for different hand sizes:
Size 1 = \( \binom{52}{1} \)
Size 5 = \( \binom{52}{5} \)
Size 10 = \( \binom{52}{10} \)
Size 13 = \( \binom{52}{13} \)

(b) How many different hands of all hearts are possible?
Number of possible hands of all hearts for different hand sizes:
Size 1 = \( \binom{13}{1} \)
Size 5 = \( \binom{13}{5} \)
Size 10 = \( \binom{13}{10} \)
Size 13 = \( \binom{13}{13} \)

(c) What is the probability of getting all hearts?
Probability of getting all hearts:
Size 1 = \( \frac{\binom{13}{1}}{\binom{52}{1}} = \frac{13}{52} = \frac{1}{4} \)
Size 5 = \( \frac{\binom{13}{5}}{\binom{52}{5}} \)
Size 10 = \( \frac{\binom{13}{10}}{\binom{52}{10}} \)
Size 13 = \( \frac{\binom{13}{13}}{\binom{52}{13}} \)

(d) What is the probability of getting all face cards (J,Q or K)?
Probability of getting all face cards
Size 1 = \( \frac{\binom{12}{1}}{\binom{52}{1}} = \frac{12}{52} = \frac{3}{13} \)
Size 5 = \( \frac{\binom{12}{5}}{\binom{52}{5}} \)
(c) What is the probability of getting all hearts or all face cards?

Probability of getting all hearts or all face cards:

Size 1 = \( \frac{13}{52} \) + \( \frac{12}{52} \) - \( \frac{3}{52} \) = 1/4 + 3/13 - 3/52 = 11/26

2. Show that the events, “draw a heart” and the event “draw a spade” are NOT independent.

- "Draw a heart" and "draw a spade" are mutually exclusive events. Both of them cannot occur at the same time. In a draw if a heart comes out then the outcome "spade" is not possible. A card cannot be heart and spade at the same time. So "draw a heart" and "draw a spade" are mutually exclusive which means occurrence of one affects the occurrence of others. So these events are not independent.

3. Consider a fair 6-sided die with the numbers 1,2,2,3,3,3 on it. For each question, you should have 3 answers.

(a) Out of 5 rolls, what is the probability of the number occurring exactly twice?

For number 1, the probability is \( \binom{5}{2}(1/6)^2(5/6)^{5-2} = 0.161 \)
For number 2, the probability is \( \binom{5}{2}(1/3)^2(2/3)^{5-2} = 0.329 \)
For number 3, the probability is \( \binom{5}{2}(1/2)^2(1/2)^{5-2} = 0.3125 \)

(b) Out of 6 rolls, what is the probability of the number occurring more than 2 times?

The probability of number occurring more than 2 times is 1 minus the probability of number occurring 0 times, 1 times and 2 times.

For number 1, the probability is 1- \( \binom{6}{0}(1/6)^0(5/6)^6-0 + \binom{6}{1}(1/6)^1(5/6)^6-1 + \binom{6}{2}(1/6)^2(5/6)^6-2 \)
For number 2, the probability is 1- \( \binom{6}{0}(1/3)^0(2/3)^6-0 + \binom{6}{1}(1/3)^1(2/3)^6-1 + \binom{6}{2}(1/3)^2(2/3)^6-2 \)
For number 3, the probability is 1- \( \binom{6}{0}(1/2)^0(1/2)^6-0 + \binom{6}{1}(1/2)^1(1/2)^6-1 + \binom{6}{2}(1/2)^2(1/2)^6-2 \)
Out of 7 rolls, what is the probability of the number occurring less than 3 times?
The probability of number occurring less than 3 times is the sum of probability of number occurring 0 times, 1 times and 2 times.

For number 1, the probability is \( \binom{7}{0}(1/6)^0(5/6)^7-0 + \binom{7}{1}(1/6)^1(5/6)^7-1 + \binom{7}{2}(1/6)^2(5/6)^7-2 \)
For number 2, the probability is \( \binom{7}{0}(1/3)^0(2/3)^7-0 + \binom{7}{1}(1/3)^1(2/3)^7-1 + \binom{7}{2}(1/3)^2(2/3)^7-2 \)
For number 3, the probability is \( \binom{7}{0}(1/2)^0(1/2)^7-0 + \binom{7}{1}(1/2)^1(1/2)^7-1 + \binom{7}{2}(1/2)^2(1/2)^7-2 \)